NANYANG JUNIOR COLLEGE

DEPARTMENT OF MATHEMATICS

H2 Mathematics **Year 1/2012**

Name: _____ **CT: 12_____**

Subject Tutor: _____ **Date: _____**

Chapter 0 Assignment

In this assignment, you are required to answer questions 1 to 26 and check your answer from the AsknLearn portal. You can consult your tutor if you have any doubts.

Q1 – 5 are multiple choice questions. You are to choose the option that best represents the correct answer out of the 5 possible options.

1. The set of non-negative integers can be represented by the set
   (A) $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ (B) $\{n \in \mathrm{Z} : n \geq 0\}$ (C) $\{n \in \mathrm{Z} : n > 0\}$
   (D) $\{n \in \mathrm{Z} : n < 0\}$ (E) Z^+

2. The value of $\sqrt{(-4)^2}$ is
   (A) 16 (B) –4 (C) 4 (D) ±4 (E) undefined

3. The expression $\dfrac{1}{x(x^2-1)(x^2+2)}$ is defined for all real values of x except
   (A) 0 only (B) ±1 only (C) ±2 only (D) 0 and ±1 only (E) 0 and ±2 only

4. For $a \neq 0$, $a^m \times a^n$ is equal to
   (A) a^{m+n} (B) a^{m-n} (C) a^{mn} (D) $a^{m/n}$ (E) none of the above

5. For $x > 0, y > 0$, $\ln\left(\dfrac{x}{y}\right)$ is the same as
   (A) $\dfrac{\ln x}{\ln y}$ (B) $\ln x + \ln y$ (C) $\ln x - \ln y$ (D) $\dfrac{\ln x}{y}$ (E) $\dfrac{x}{\ln y}$

Q6 – 11 are statements which are either true or false. State whether each of the following statements is true or false. If a statement is false, correct it to make it true.

6. For $a > 0$, $(a^m)^n = a^{mn}$.

7. $\ln x^2 = 2\ln x$ for all real values of x.

8. For $x > 0, y > 0$, $x > y \Leftrightarrow \ln x > \ln y$.

9. $x > y \Rightarrow x^2 > y^2$ for all real values of x and y.

10. $\sqrt{x^2} = x$ for all real values of x.

11. $x^2 = y^2 \Rightarrow x = y$ for all real values of x and y.

Q12 – 26 are questions that require working. Show your workings clearly.

12. Given that –4 is a root of the equation $x^3 + 4x^2 + kx - 4 = 0$, find the value of k and hence solve the equation completely.

13. Show that for all $x \in \mathrm{R}$, $4x^2 + 4x$ cannot be less than a certain value and find this value. Deduce the range of values of c such that the line $y = c$ cuts the graph of $y = 4x^2 + 4x$ at exactly 2 points.

14. Find the range of values of k for which $kx^2 + 8x > 6 - k$ for all real values of x.

15. Express $\dfrac{1}{x^3 - 2x^2 + x - 2}$ as a sum of partial fractions.

16. Find the solution set of the following inequalities:
(a) $\ln(x+1) \geq \ln 2$ (b) $\ln x < 1$

17. Prove that for all $x > 0$, $\mathrm{e}^{\ln x} = x$.

18. Solve the following equations:
(a) $|x-1| = 2$ (b) $\mathrm{e}^{x^2-4} = 1$ (c) $\mathrm{e}^x = \dfrac{1}{2}$ (d) $\ln x^2 = 4$

19. Write down the exact value of each of the following:
(a) $\sin 240°$ (b) $\tan(-60°)$ (c) $\cos\dfrac{5\pi}{3}$ (d) $\sin n\pi$ ($n \in \mathrm{Z}$) (e) $\cos n\pi$ ($n \in \mathrm{Z}$)

20. Given that $\sin x = \dfrac{1}{2}$ where $\dfrac{\pi}{2} < x < \pi$, find, without the use of a calculator, the exact values of $\cos x$, $\tan x$, $\sin 2x$ and $\tan 2x$.

21. Solve the following trigonometric equations:
(a) $\sin 2x = \dfrac{\sqrt{3}}{2}$ $(0 < x \leq 2\pi)$ (b) $\tan(x - 30^\circ) = 1$ $(-180^\circ < x < 180^\circ)$

22. Solve the following trigonometric inequalities for $0 \leq x \leq \pi$:
(a) $\tan 2x \leq \sqrt{3}$ (b) $\cos 2x > \dfrac{1}{2}$

23. In a right-angled triangle ABC, $\angle B = 90^\circ$ and $\angle A = \alpha^\circ$. Show that $\sin(90^\circ - \alpha^\circ) = \cos\alpha^\circ$. Hence deduce the exact value of $\dfrac{\cos 20^\circ}{\cos 20^\circ + \sin 70^\circ}$.

24. Prove that $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$. Deduce that $\sin 15^\circ \sin 75^\circ = \dfrac{1}{4}$.

25. Prove the following trigonometric identities:
(a) $\cos^4 x = \dfrac{1}{8}(3 + 4\cos 2x + \cos 4x)$ (b) $\dfrac{\sin x}{\sqrt{(1-\cos x)}} = \sqrt{2}\cos\dfrac{1}{2}x$ $(0 < x < \dfrac{\pi}{2})$

26. Solve the equation $\sin x + \sin 2x + \sin 3x = 0$ for $0^\circ < x < 360^\circ$.

Q27 – 29 may be attempted after you have studied the topic "Functions".

27. Evaluate $\cos\left[\tan^{-1}\left(-\frac{4}{3}\right)\right]$ exactly.

28. If $y = \sin^{-1} x$, prove that $\sec y = \frac{1}{\sqrt{1-x^2}}$.

29. Given that $\theta = \cos^{-1}\left(\frac{1}{4}x\right)$ where $x > 0$, find, in terms of x,

    (i) $\sin\theta$, (ii) $\cot\theta$, (iii) $\sin 2\theta$

Q30 – 35 are optional questions for the interested student

30. Prove that the roots of $ax^2 + bx - a^3 = 0$ ($a \neq 0$) cannot have the same sign. If α and β are the roots of the above equation, find $\alpha^2 + \beta^2$ in terms of a and b.

31. If a, b and c are odd integers, prove that $ax^2 + bx + c = 0$ cannot have rational solutions.

32. Prove that the only solutions of the equation $\cos^{10}x - \sin^{10}x = 1$ are of the form $n\pi$ where $n \in \mathrm{Z}$.

33. In triangle ABC the bisector of angle A meets BC at D.
    Use the sine rule in triangles ABD and ACD to show that $\frac{BD}{DC} = \frac{BA}{AC}$.
    Given that the triangle ABC is equilateral, so that D is the mid-point of BC, and that the bisector of angle BAD meets BD at E, show that $\frac{ED}{BD} = \frac{\sqrt{3}}{2+\sqrt{3}}$.

34. Prove that if α, β, γ are angles of a triangle, then
    $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 - 2\cos\alpha\cos\beta\cos\gamma.$$

35. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$. Hence find the positive value of x given that $\cos^{-1}x + \cos^{-1}\sqrt{3}\,x + \cos^{-1}\left(x - \frac{1}{2}\right) = \pi$.